$\boxed{\text{Sec 1}}$

sheet 1 Math 4

$\boxed{1}$ $\varphi = xy^2 + yz^3$ at point $(2,-1,1)$

parallel vector $\vec{i} + 2\vec{j} + 2\vec{k}$

Sol.

$$D_{\vec{a}}\varphi(x,y,z) = \bar{\nabla}\varphi\Big|_p \cdot \frac{\vec{a}}{\|\vec{a}\|} = (1,-3,-3)\cdot\frac{(1,2,2)}{3} = \frac{-11}{3}$$

$$\bar{\nabla}\varphi = y^2\vec{i} + (2xy + z^3)\vec{j} + (3yz^2)\vec{k}$$

$$\bar{\nabla}\varphi\Big|_{(2,-1,1)} = \vec{i} - 3\vec{j} - 3\vec{k} = (1,-3,-3)$$

$\boxed{4}$ find $\bar{\nabla}\varphi$ $\quad \varphi = xyz \quad \varphi = (x^2+y^2+z^2)^{-\frac{1}{2}}$

$$\bar{\nabla}\varphi = yz\vec{i} + xz\vec{j} + xy\vec{k}$$

$$\bar{\nabla}\varphi \cancel{= \frac{-1}{2}(2x)^{-\frac{3}{2}}\vec{i} + \frac{-1}{2}(2y)\vec{j} + \frac{-1}{2}(2z)\vec{k}}$$

$$\bar{\nabla}\varphi = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{3}{2}}\cancel{2}x - \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{3}{2}}\cancel{2}y$$

$$-\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{3}{2}}\cancel{2}z$$

$$= -x(x^2+y^2+z^2)^{-\frac{3}{2}} - y(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$-z(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$\varphi = x^2 + y^2 + z^2$ point $(1,2,3)$

stright Line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

$\bar{\nabla}\varphi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$

at point $(1,2,3)$

$\vec{\nabla}\varphi = 2\vec{i} + 4\vec{j} + 6\vec{k}$

$D_{\bar{a}}\varphi = (2,4,6)\left(\frac{(3,4,5)}{5\sqrt{2}}\right)$

$= \frac{52}{5\sqrt{2}}$

from paramitric eq.

$x = x_0 + a_x t$

$y = y_0 + a_y t$

$z = z_0 + a_z t$

symmetric eq.

$\therefore \frac{x - x_0}{a_x} = \frac{y - y_0}{a_y} = \frac{z - z}{a_z}$

$\therefore \vec{a} = (3,4,5)$

$\|a\| = \sqrt{9+16+25} = 5\sqrt{2}$

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9) The unit vector normal to the Surface $x^2y^2z^3 = 8$ at The point $(1,1,2$

$\varphi = x^2y^2z^3 - 8$

$\bar{\nabla}\varphi = 2xy^2z^3\vec{i} + 2yx^2z^3\vec{j} + 3z^2x^2y^2\vec{k}$

at $(1,1,2)$

$= 16\vec{i} + 16\vec{j} + 12\vec{k}$

$\hat{n} = \frac{\nabla\varphi}{\|\nabla\varphi\|}$

$\|\nabla\varphi\| = \sqrt{16^2 + 16^2 + 12^2}$

$= 4\sqrt{41}$

$\vec{n} = \frac{16}{4\sqrt{41}}\vec{i} + \frac{16}{4\sqrt{41}}\vec{j} + \frac{12}{4\sqrt{41}}\vec{k}$

$= \frac{4}{\sqrt{41}}\vec{i} + \frac{4}{\sqrt{41}}\vec{j} + \frac{3}{\sqrt{41}}\vec{k}$

(8) $u = x+y+z \quad v = x+y \quad w = -2xz - z^2 - 2yz$

prove The triple scalar product
$[\bar{\nabla} u, \bar{\nabla} v, \bar{\nabla} w] = 0$

$\bar{\nabla} u = \vec{i} + \vec{j} + \vec{k} \qquad \bar{\nabla} v = \vec{i} + \vec{j}$

$\bar{\nabla} w = -2z\vec{i} \cancel{+} -2z - 2z\vec{k}$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ -2z & -2z & \cancel{-2z} \end{vmatrix} = \cancel{1(-2z)} \cancel{+} \cancel{(-2z)}$$

$-2y-2x-2z$

$= (-2y-2x-2z) - (-2y-2x-2z)$

$(-2z) - (-2z) = 0$